Column Generation for Dimensioning Resilient Optical Grid Networks with Relocation

Brigitte Jaumard*, Jens Buysse†, Ali Shaikh*, Marc De Leenheer† and Chris Develder†
* CSE Dpt, Concordia University
Montreal (Qc) Canada
Email: bjaumard@ciise.concordia.ca
† Ghent University – IBBT, INTEC
Ghent, Belgium
Email: jens.buysse, chris.develder@intec.ugent.be

***Abstract*—Nowadays, the Quality of Service (QoS) in Optical Grids has become a key issue. An important QoS factor is the resiliency, namely the ability to survive from certain network failures. Although several traditional network protection schemes were devised in the past, they are not optimized for Optical Grids.**

In an earlier work, we proposed relocation strategies providing backup paths to alternate destinations, exploiting the anycast routing principle of grids. To show the advantage of relocation (compared to traditional network protection) in terms of reduced network capacity, we formulated the network dimensioning problem as an Integer Linear Program (ILP). Yet, its solution exhibited very poor scalability and appeared not practical for reasonably large scaled case studies.

Therefore, we propose a novel formulation for the relocation protection scheme, using column generation (CG). This approach decomposes the original ILP into two parts, specifically a Restricted Master Problem (RMP) and a Pricing Problem (PP) which are iteratively and alternatively solved until the optimality condition is satisfied. Such a CG decomposition has a significant impact on the complexity of the model, leading to a significant improvement over previous ILPs in term of scalability and running times. We demonstrate that the CG method is highly scalable and generates nearly optimal solutions using case studies with up to 300 connections, showing it to be highly competitive with a previously proposed heuristic. We also perform some comparisons of the anycast scheme with the classical shared path protection on larger network instances.

I. Introduction

Grids are a form of distributed computing where several computational and data resources are coupled together in order to execute highly demanding tasks. These emerging applications require predictable services and on-demand-data delivery. Consequently, a network supporting a Grid network should be able to bear large data transfers in a fast and reliable way. Given their high data rates and low latency, optical networks based on wavelength division multiplexing (WDM) technology are ideally suited to support the Grid networks, thus giving rise to so-called optical grids [1].

A major concern in deploying optical grids is resiliency: ensuring service continuity under failure conditions is of utmost importance. To deal with potential network failures, various network resilience strategies for WDM networks have been devised (for an extensive overview, see [2] [3]). For instance, end-to-end (or path) protection schemes have been developed protecting against single link failures: a primary path is protected by a link-disjoint backup path which is used in case of link failure (this link diversity guarantees that the primary and backup path will never fail simultaneously for any single link failure). This corresponds to the framework of Classical Shared Path Protection (CSP). These protection strategies can however be optimized for the optical grid scenario, by exploiting the anycast routing principle typical of grid scenarios. There, a user submitting a job only cares about timely and correct processing of his job, but is indifferent about the location of the execution of it. So, instead of reserving a backup path to the resource indicated by the Grid scheduler under failure-free conditions, it could be better to relocate the job to another resource if this implies network resource savings. For example, a backup path to the primary server could cross a server location. This corresponds to the so-called Shared Path Protection with Relocation (SPR).

In [4], we have considered the impact of using the relocation mechanism for the case of shared path protection, i.e. we allow the wavelengths used for backup paths to be shared among multiple backup paths as long as the corresponding primary paths are link disjoint. In small-scale case studies, it was shown that the bandwidth savings (i.e., reduced number of provisioned wavelengths) achieved by relocation compared to traditional shared path protection amounts to 20%. The reason for the small scale studies in that work is that the dimensioning problem solved is not scalable to larger scale scenarios: the classical Integer Linear Programming (ILP) solutions are known to scale poorly as soon as the instances are getting larger.

To address this scalability problem, we propose to use a column generation (CG) approach [5] for modeling and solving the problem. Indeed, CG models have already been successfully used for solving several design and management problems in optical and wireless networks, e.g., p-cycle based problems [6] [7] and resource allocation in WiMax networks [8].

The paper is organized as follows. In Section II, we present the classical ILP formulation [4] which proved to perform badly when applied on larger network instances. We continue in Section III with the new column generation model where we explain how to create and solve the CG model. We conclude in Section IV with the description of a heuristic to solve the resilient optical grid dimensioning problem [9]. The last Section V, is devoted to a case study and comparisons of performance of the two ILP models and the heuristic where we conclude that both the CG model and the heuristic scheme are very efficient, scalable and generate comparable solutions for large traffic instances.

II. The classical ILP formulation

We first investigate two ILP models for dimensioning resilient optical grids assuming a relocation strategy. The traffic model is such that each connection represents a point-to-point connection between a source and a destination. Furthermore, we assume that all optical cross-connects (OXC) are able to perform wavelength conversion. The network and traffic instances are described by the following parameters:

- G — $= (V, L)$, a graph representing an optical grid
- V — Node set, indexed by $v \in V$
- V_b — $\subset V$, a set of nodes with resources (i.e. grid server sites).
- L — Link set, indexed by $\ell \in L$
- K — Request set, indexed by $k \in K$
- K_{sd} — Request set for the requests from v_s to v_d, indexed by $k \in K_{sd}$
- D_{sd} — $= |K_{sd}|$, i.e., number of unit demands between source v_s and destination v_d
- $\mathcal{SD}$ — Set of pairs (v_s, v_d) such that $D_{sd} > 0$
- CAP_ℓ — for all $\ell \in L$, transport capacity of link $\ell \in L$. We will assume all links have the same transport capacity (in terms of wavelengths), say $\text{CAP}_\ell = \text{CAP}$ for all $\ell \in L$.

The ILP Model that is presented below is fairly similar to [4], except for the notations, which we simplified and made consistent with the column generation model of the next section.

The variables are as follows.

- w_ℓ^k — binary variable which is equal to 1 if request k is routed (working path) through ℓ, 0 otherwise.
- p_ℓ^k — binary variable which is equal to 1 if request k is routed (backup path) through ℓ, 0 otherwise.
- CAP_ℓ^P — $\in \mathbb{Z}^+$. Integer variable that is equal to the number of shared backup wavelengths on link ℓ.
- b_v^k — decision variable that is equal to 1 if v is used as backup resource for connection k.
- $\text{CAP}_{k\ell\ell'}^P$ — $\in \mathbb{Z}^+$. Integer variable to help count the backup wavelengths.

The objective function (1) aims at minimizing the overall network capacity:

$$\min \quad \sum_{\ell \in L} \left(\text{CAP}_\ell^P + \sum_{k \in K} w_\ell^k \right). \tag{1}$$

The first set of constraints define the demand constraints and the flow conservation constraints for the primary paths:

$$\sum_{\ell \in \omega^+(v)} w_\ell^k - \sum_{\ell \in \omega^-(v)} w_\ell^k = \begin{cases} -1 & \text{if } v = v_s \\ +1 & \text{if } v = v_d \\ 0 & \text{otherwise} \end{cases}$$
$$v \in V, k \in K_{sd}, (v_s, v_d) \in \mathcal{SD}. \tag{2}$$

The next set of constraints expresses the demand constraints and flow conservation constraints for the backup paths:

$$\sum_{\ell \in \omega^+(v)} p_\ell^k - \sum_{\ell \in \omega^-(v)} p_\ell^k = \begin{cases} -1 & \text{if } v = v_s \\ b_v^k & \text{if } v \in V_B \\ 0 & \text{otherwise} \end{cases}$$
$$v \in V, k \in K_{sd}, (v_s, v_d) \in \mathcal{SD}. \tag{3}$$

Then, we must ensure that pairs of working and backup paths do not overlap:

$$w_\ell^k + p_\ell^k \le 1 \qquad \ell \in L, k \in K. \tag{4}$$

We next calculate the capacities on link ℓ:

$$\text{CAP}_{\ell'}^P \ge \sum_{k \in K} \text{CAP}_{k\ell\ell'}^P \qquad \ell, \ell' \in L : \ell \ne \ell' \tag{5}$$

$$\text{CAP}_{k\ell\ell'}^P \ge w_\ell^k + p_{\ell'}^k - 1 \qquad k \in K;\ \ell, \ell' \in L : \ell \ne \ell' \tag{6}$$

If we consider Classical Shared Path Protection (CSP), we have to set the b_v^k variables to 1 if v is the primary server of connection k:

$$b_v^k = \begin{cases} 1 & v \text{ is the primary server of } k \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

On the other hand, if we consider Shared Path Protection with Relocation (SPR), constraints (7) have to be replaced with constraints (8) in order to allow each backup server to be different from each primary server:

$$\sum_{v \in V_B} b_v^k = 1 \qquad k \in K. \tag{8}$$

III. Column Generation ILP Model

A. Creating and solving the CG model

The philosophy of Column Generation is to limit the number of variables explicitly included in the ILP problem. This amounts to leaving out columns in an implicit matrix from of the LP. This reduction of the problem size is motivated by the fact that the values of the associated variables are zero in the optimal solution (i.e. there are non-basis variables). CG corresponds to an iterative procedure where columns are added one at a time and only if their addition allows reducing the value of the cost objective function. Hence, the original cost minimization problem is split into two sub-problems: a Restricted Master Problem (RMP) and a Pricing Problem (PP). The RMP is a restricted version of the original problem as it only contains a subset of the original columns. This RMP needs to be solved optimally after which we formulate a reduced cost function which serves as the objective function of the second sub-problem, so-called Pricing Problem (PP). PP needs to be minimized under several constraints i.e. the constraints defining the relations among the coefficients of a column. If the reduced cost solution is less than zero, it means we have identified a variable whose addition in the RMP will increase the objective value and the RMP needs to be solved again with the added variable (and associated column). If PP has no solution with a negative reduced cost, the current LP solution is indeed the optimal solution of the LP relaxation of the problem. All what remains, is to derive an ILP (Integer Linear Program) solution.

Getting an ILP solution when the LP relaxation is solved using column generation requires a branch-and-price method in order to get an optimal solution, see e.g. [10]. However, it is usually quite costly in terms of computational time, and very often only heuristics are sought after, with the information on how far the heuristic solution is from the optimal one, throughout the so-called optimality gap, i.e. the difference between the values of the heuristic ILP solution and the optimal LP relaxation value. Popular heuristics are solving the ILP made of the generated columns in order to reach the optimal LP solution and rounding off methods, [10], [7]. In this study, we use the first heuristic approach, and got optimality gaps less than 1%, i.e., very acurate near optimal ILP solutions.

B. CG specific parameters

- c A configuration. It is defined for a given pair of source and destination nodes (v_s, v_d), and consists of a working path from v_s to v_d, and a protection path either from v_s to v_d, or to v_s to a node $v \in V_B$.
- C_{sd} Set of configurations associated with the pair (v_s, v_d)
- C $= \bigcup_{(v_s,v_d)\in\mathcal{SD}} C_{sd}$
- w^c_ℓ = 1 if link ℓ is used by the working path in configuration $c \in C_{sd}$, 0 otherwise.
- p^c_ℓ = 1 if link ℓ is used by the protection path $c \in C_{sd}$, whether the protection goes to v_d or to an alternate destination node $v \in V_B$, 0 otherwise.

C. The Master Problem

The only set of variables which need to be optimized are the $z^c \in \mathbb{Z}^+$ and $cap^c_\ell \in \mathbb{Z}^+$. Each z^c represents the number of selected copies of configuration c. The w^c_ℓ and p^c_ℓ are the parameters.

The objective function which minimizes the total network capacity, can be written as follows:

$$\min \quad \sum_{\ell\in L}\left(cap^P_\ell + \sum_{(v_s,v_d)\in\mathcal{SD}}\ \sum_{c\in C_{sd}} w^c_\ell\, z^c\right) \tag{9}$$

Demand constraints are written as follows:

$$\sum_{c\in C_{sd}} z^c \geq D_{sd}. \qquad (v_s,v_d)\in\mathcal{SD} \tag{10}$$

The next set of constraints express the capacity requirement for a link ℓ' in a backup path. Indeed, if ℓ' protects link ℓ, with ℓ belonging to several working paths (modeled here throughout the various configurations associated with working paths containing ℓ), we must ensure ℓ' to have a large enough transport capacity:

$$\sum_{c\in C} w^c_\ell\, p^c_{\ell'}\, z^c \leq \mathrm{CAP}^P_{\ell'} \qquad \ell,\ell'\in L : \ell' \neq \ell. \tag{11}$$

D. The Pricing Problem

The second component in the decomposition induced by the column generation model corresponds to the so-called pricing problem. When solving the pricing problem, we either find a new configuration which when added to the RMP will improve the current cost of the RMP, or we will be able to conclude that we have reached the optimal value of the LP relaxation. The number of pricing problems equals the number of pairs of source and destination nodes.

Let us denote the dual vectors (which are the parameters of the PP) as follows:

- u^1_{sd} dual vector of constraint (10)
- $u^2_{\ell\ell'}$ dual vector of constraint (11)

The objective function of the PP for demand pair (v_s, v_d) of source and destination nodes, corresponds to the minimization of the reduced cost function:

$$\overline{\mathrm{COST}}_{sd}(w,p,b) = \sum_{\ell\in L} w_\ell - u^1_{sd} - \sum_{\ell\in L}\ \sum_{\ell'\in L:\ell\neq\ell'} u^2_{\ell\ell'} w_\ell p_{\ell'} \tag{12}$$

The set of constraints is similar to the one of the classical ILP in II, except that the constraints only apply for a given pair of source and destination nodes (i.e. configuration c).

$$\sum_{\ell\in\omega^+(v)} w^c_\ell - \sum_{\ell\in\omega^-(v)} w^c_\ell = \begin{cases} -1 & \text{if } v = v_s \\ +1 & \text{if } v = v_d \\ 0 & \text{otherwise} \end{cases} \qquad v \in V \tag{13}$$

$$\sum_{\ell\in\omega^+(v)} p^c_\ell - \sum_{\ell\in\omega^+(v)} p^c_\ell = \begin{cases} -1 & \text{if } v = v_s \\ b_v & \text{if } v \in V_B \\ 0 & \text{otherwise} \end{cases} \qquad v \in V \tag{14}$$

$$w^c_\ell + p^c_\ell \leq 1 \qquad \ell \in L \tag{15}$$

If we consider CSP, we need the following constraints:

$$b_v = \begin{cases} 1 & v \text{ is the primary server of } c. \\ 0 & \text{else,} \end{cases} \tag{16}$$

while, if we consider SPR, we replace constraints (16) with the following constraints:

$$\sum_{v\in V_B} b_v = 1 \tag{17}$$

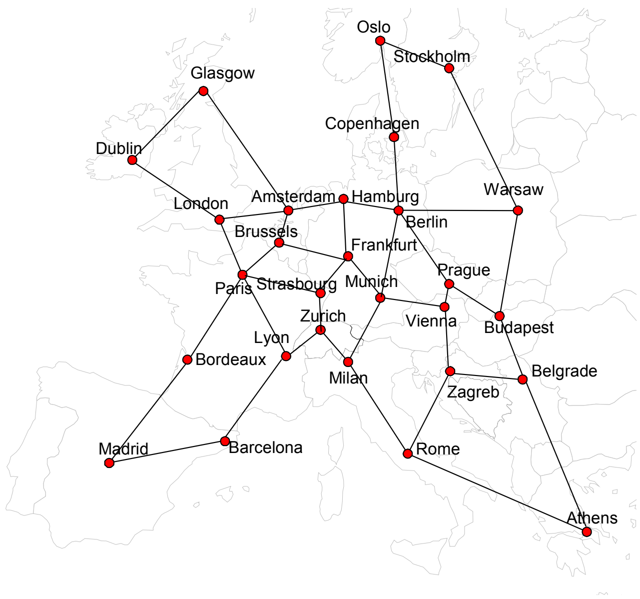


Fig. 1. European network

As can be observed, the expression of the reduced cost (12) is nonlinear. In order to reduce to a linear integer programming model, we introduce the following set of variables:

$$wp_{\ell\ell'} = w_\ell p_{\ell'} \in \{0,1\} \qquad \ell, \ell' \in L : \ell \neq \ell' \tag{18}$$

The expression of the objective (i.e., reduced cost) of the pricing problem becomes:

$$\overline{\text{COST}}(w) = \sum_{\ell \in L} w_\ell - u^1_{sd} - \sum_{\ell \in L} \sum_{\ell' \in L : \ell \neq \ell'} u^2_{\ell\ell'} wp_{\ell\ell'} \tag{19}$$

We then need to add the following linearization constraints to enforce the relation (18) between the variables w_ℓ, $p_{\ell'}$ and $wp_{\ell\ell'}$:

$$wp_{\ell\ell'} \geq w_\ell + p_{\ell'} - 1 \qquad \ell, \ell' \in L : \ell \neq \ell'. \tag{20}$$

Note that if we want to reinforce link capacity constraints in the CG model, we need to add the following constraints:

$$\sum_{c \in C} w^c_\ell \, z^c + \text{CAP}^P_\ell \leq \text{CAP}_\ell \qquad \ell \in L \tag{21}$$

in the master model and change pricing constraints as follows.

$$\overline{\text{COST}}_{sd}(w, p, b) = \sum_{\ell \in L} w_\ell - u^1_{sd} + \sum_{\ell \in L} \sum_{\ell' \in L : \ell \neq \ell'} u^2_{\ell\ell'} w_\ell p_{\ell'} - \sum_{\ell \in L} u^3 w_\ell, \tag{22}$$

where u^3 is the dual vector associated with (21).

IV. Heuristic

As an alternative to the above linear programming solutions, we also devised a heuristic to solve the dimensioning problem for both CSP and SPR cases. We adapted the one presented in [9], where traffic is described by a demand vector that only specifies the sources and lets the heuristic determine the destinations. In the current work, we assume a static traffic matrix with both the source and destination nodes present(for the primary paths). The objective of the heuristic is again to minimize the overall required capacity, i.e. to minimize the required number of wavelengths for both primary (by limiting path length) and backup (by maximizing sharing) connections. It proceeds in two stages: (i)

1) *Initialization:* Find a pair of link disjoint paths from the source to the destination with minimum cost. (In the SPR case, the primary and backup resources are not necessarily identical.)
2) *Optimization:* Find a new configuration (i.e., the combination of a primary and a backup path) which minimizes the total network utilization by rerouting both. In most circumstances, the primary path does not get altered, but the backup path does get rerouted over links which are part of another backup path, protecting a link disjoint primary path. In the SPR case, we also try to change the backup resource and see if we cannot find a better configuration.

For more details about the heuristic, we refer the reader to [9].

V. Case Study

We have considered an European topology, comprising 28 nodes and 40 bidirectional links, as depicted in Fig. 1. We have chosen 5 nodes as Grid server resource sites: Dublin, Paris, Zurich, Munich and Berlin.

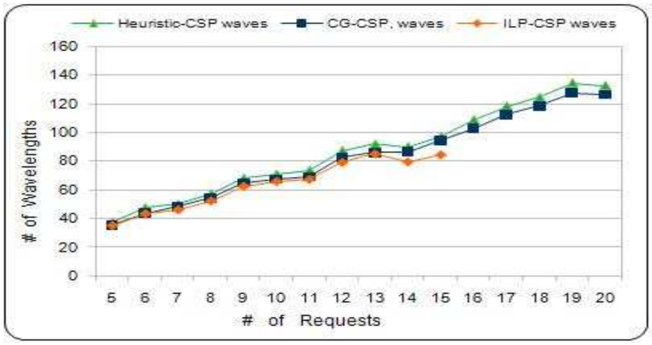
To evaluate the solutions of the classical ILP and CG models, together with the heuristic solution, we have randomly generated multiple demand matrix instances with a varying number of connections. The results described in this section correspond to average values on a set of 11 instances.

A. Comparative Performances

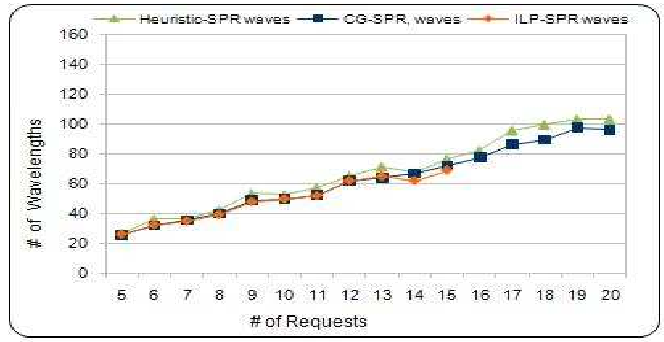
Before we go into the details of the comparative performances of the different methods on large traffic instances, we first evaluate the results on smaller instances. In Figure 2(a) and Figure 2(b), we plotted the total number of wavelengths which the different methods output for the optimized capacity value, with a number of requests varying from 5 to 20. In all plots, ILP refers to the classical ILP model, CG to the CG ILP model, H to the heuristic described in Section IV, while CSP and SPR refers to Classical Shared Path Protection and Shared Path Protection with Relocation respectively.

For the demand set with more 9 than demands, difficulties started to appear when solving the classical ILP model in a reasonable time frame. Indeed, out of the 11 solved instances, there were always one or two instances which could not be solved within the 72 hours time limit we set ourselves. This is why for demand set with more than 15 requested connections, we did not use the classical ILP model anymore.

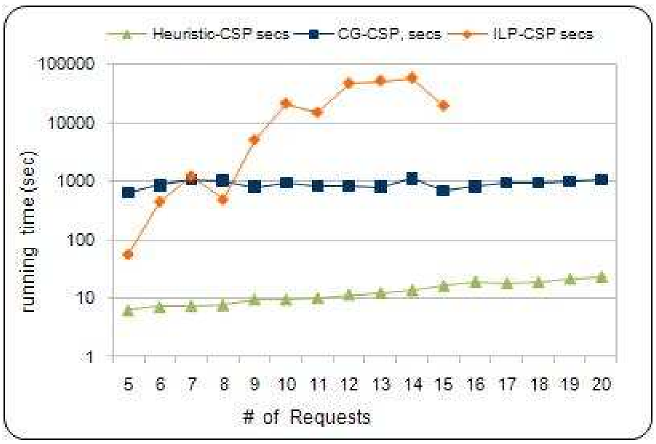
We observe a small optimality gap for CG, comparing it to the exact ILP solution. The heuristic performs quite well also, but worse than CG. We calculated the average gap for



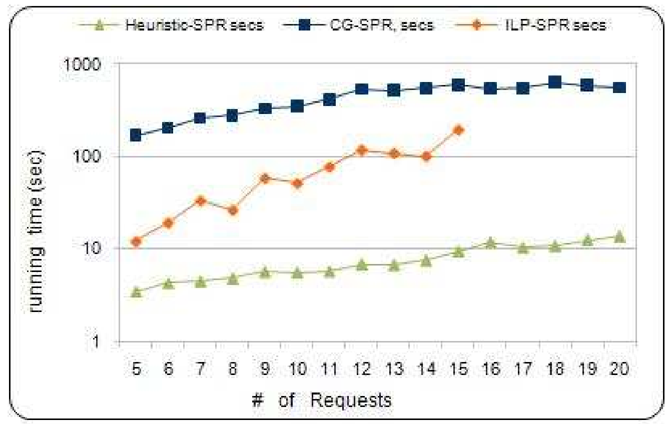
(a) CSP : Total number of wavelengths.



(b) SPR: Total number of wavelengths

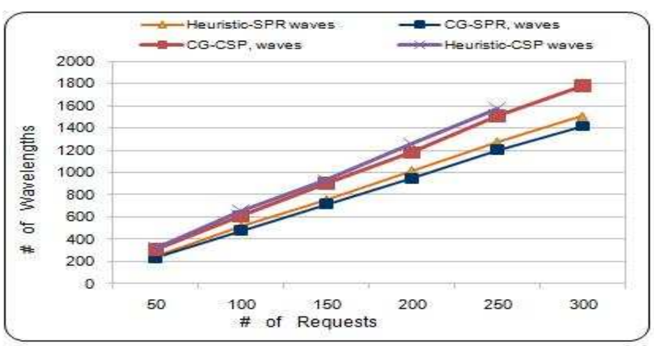


(c) CSP: execution time

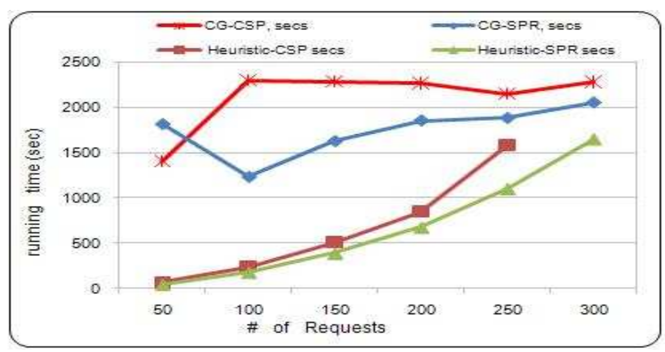


(d) SPR: execution time

Fig. 2. The results obtained for cases with a demand from 5 to 20. (Note the different scales for the execution times of CSP and SPR.)



(a) Total number of wavelengths



(b) Execution time of methods

Fig. 3. Results obtained for cases with a demand from 50 to 300.

the request range [5, 13] (since the ILP average does not include all 11 instances for demands beyond 13 requests). With respect to the comparison between the two protection schemes, there is only an optimality gap of 2.46% in the CSP and 1.11% in the SPR case if we compare ILP with CG. The heuristic generates inferior results compared to CG: on average a difference of 8.63% for CSP, 8.15% for SPR. Comparing the results generated by the CG method and the heuristic, we come to the conclusion that the gap between them remains fairly constant: for CSP a difference of 5.48% and for SPR 5.71%. This lead us to the conlusion that the CG has an output which estimates the optimal output very well and the heuristic has suboptimal solutions, which are still acceptable.

The trend is fairly similar in 3(a) where we plotted the total number of wavelengths for the demand sets with 50 to 300 requested connections. We ascertain that the difference between the total number of wavelengths for the heuristic and CG averages to 4.99% for the CSP case and 6.92% for SPR.

As a last observation, note that the conclusions made in [4] and [9] are confirmed are large traffic instances: relocation impacts the network dimension by introducing a network load reduction (NLR). Here, it amounts to $\pm 22\%$, independently of the requested number of connections.

B. Computational Effort

Scalability is a known issue in planning problems solved by ILPs. This is our main motivation for using other techniques such as heuristics and decomposition column generation techniques. In Fig. 2(c) and Fig. 2(d), we compare the computational times (in seconds) of the three methods.

Firstly, we observe that up to 9 requests, the classical ILP model performs better than the CG model. This can be explained by the initial phase of the CG solution which is heavier than in the classical ILP. The CG model does not have a “warm” start: we begin with an empty matrix and slowly add one column at a time, building a feasible solution with the drawback that as long as a critical number of columns has not yet been generated (at least as many as the number of constraints), dual values are not as meaningful. Therefore, generated columns are usually not the ones that will be part of the optimal solution with a nonzero value for the associated variables. In addition, as there are as many pricing problems as the number of pairs of origin and destination nodes and as we use a classical round robin procedure to regularly solve them, there is a cost in terms of the number of iterations before reaching the optimal solution of the LP solution.

Results on larger traffic instances show that the heuristic generates very good solutions, however with a computational cost that is comparable with the solution of the CG model. It therefore shows that, on the one hand, CG model offers a fairly scalable tool even for large ILP models, and on the other hand that there is a trade off in heuristics between the computational time and quality of the solutions: accurate solutions maybe costly to reach with a heuristic when the problem to be solved is quite combinatorial in nature.

VI. Conclusion

We have developed a new scalable ILP model based on a column generation method that allows investigating further the SPR protection scheme in optical grids. By splitting the traditional ILP formulation into a Restricted Master Problem and a Pricing Problem, column generation is able to handle large network instances (which the traditional ILP could not cope with). Numerical results from a study on an European Network have shown that CG leads to a very good approximation of the optimal result. When comparing the CG method to the proposed heuristic, we see that although the estimation of the result with CG is better, the execution time of the heuristic is favorable. But this does not hold anymore for larger instances where we notice that the computational effort for the heuristic and CG is similar.

Future work includes the improvement of the CG ILP and there are different avenues along which we walk e.g., warm start, better exploration scheme of the different pricing problems. It also allows thinking of additional constraints to be taken into account in the efficient dimensioning of resilient optical grids, e.g., with respect to QoS constraints. We can also investigate the impact of the number of server sites on the network dimensions (and savings by SPR), as in [1] without resilience.

Acknowledgment

The first author has been supported by a Concordia University Research Chair (Tier I) and by an NSERC (Natural Sciences and Engineering Research Council of Canada) grant. For the authors from Belgium, the work described in this paper was carried out with the support of the BONE-project (Building the Future Optical Network in Europe), a Network of Excellence funded by the European Commission through the 7th ICT-Framework Programme. C. Develder and M. De Leenheer are supported by the Research Foundation – Flanders (FWO) as post-doctoral fellows. J. Buysse is supported by a PhD grant of the Flemish government agency for Innovation by Science and Technology (IWT).

References

[1] C. Develder, B. Mukherjee, B. Dhoedt, and P. Demeester, “On dimensioning optical grids and the impact of scheduling,” *Photonic Netw. Commun.*, vol. 17, no. 3, pp. 255–265, Jun. 2009.

[2] A. Concaro, G. Maier, M. Martinelli, A. Pattavina, and M. Tornatore, “Qos provision in optical networks by shared protection: An exact approach,” in *Quality of Service in Multiservice IP Networks: Second International Workshop QoS-IP, Milano, Italy*, 2003, pp. 419–432. [Online]. Available: http://www.springerlink.com/content/qlbjra5hw5kexy5a

[3] S. Ramamurthy and B. Mukherjee, “Survivable WDM mesh networks - part i. protection,” in *INFOCOM*, vol. 2, 1999.

[4] J. Buysse, M. D. Leenheer, C. Develder, and B. Dhoedt, “Exploiting relocation to reduce network dimensions of resilient optical grids,” in *Proceedings of IEEE/VDE Workshop on Design of Reliable Communication Networks - DRCN*, October 2009.

[5] V. Chvatal, *Linear Programming*. Freeman, 1983.

[6] S. Sebbah and B. Jaumard, “Efficient and scalable design of protected working capacity envelope,” in *13th International Telecommunications Network Strategy and Planning Symposium - Networks*, 2008, pp. 1 – 21.

[7] B. Jaumard, C. Rocha, D. Baloukov, and W. D. Grover, “A column generation approach for design of networks using path-protecting p-cycles,” in *Proceedings of IEEE/VDE Workshop on Design of Reliable Communication Networks - DRCN*, October 2007.

[8] A. Capone and G. Carello, “Scheduling optimization in wireless mesh networks with power control and rate adaptation,” in *IEEE SECON 2006*, 2006, pp. 138–147.

[9] J. Buysse, M. D. Leenheer, B. Dhoedt, and C. Develder, “On the impact of relocation on network dimensions in resilient optical grids,” in *Optical Network Design and Modeling - ONDM*, February 2010.

[10] C. Barnhart, E. Johnson, G. Nemhauser, M. Savelsbergh, and P. Vance, “Branch-and-price: Column generation for solving huge integer programs,” *Operations Research*, vol. 46, no. 3, pp. 316–329, May-June 1998.